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WAVE PROPAGATION IN MAGNETO-THERMO ELASTIC SOLIDS IN THE PRESENCE OF STATIC STRESSES

Manjula Ramagiri
Department of Mathematics, University Arts and Science College, Kakatiya University, Warangal, Telangana

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Abstract:

The present paper is consent with the investigation of wave propagation in a homogeneous isotropic and thermo elastic medium with magnetic field and a time dependent heat source effect due to thermo mechanical source. The governing equations is solved by Lames constants and obtained the frequency equation which determines the phase velocity, static stress, magnetic field has been investigated. Numerical results analyzing the frequency equation are discussed and presented. The results indicate that the phase velocity increases with increase in wave number and uniaxial static stress.

Key Words: Thermo Elasticity, Magnetic Field, Static Stress, Phase Velocity, Wave Number

1. Introduction:

The wave proapagation in magneto-thermo elastic solids is important in many fields such as earthquake engineering, soil dynamics, nuclear reactors, high energy particles accelerators etc. On propagation of thermo elastic waves in homogeneous isotropic plates is studied in [1]. In this paper they discussed that the phase velocities of symmetric and skew symmetric modes of wave propagation is computed for various values of wave number. In [2] authors studied the propagation of magneto-thermo elastic plane waves. In the said paper, the analysis is essentially a reconciliation of the governing equations in three fields such as the electromagnetic field, the thermal field and the elastic field, which interact one with another. Electro-magneto thermo elastic plane waves with thermal relaxation in a medium of perfect conductivity is discussed in [3]. In this they discussed three different methods. The first deals with a thick plate of perfect conductivity subjected to a time-dependent heat source on each face; the second concerns the case of a heated punch moving across the surface of a semi-infinite thermo elastic half-space of perfect conductivity subject to appropriate boundary conditions; and the third problem deals with a plate with thermo-isolated surfaces subjected to time-dependent compression. Reflection of plane waves in a rotating transversely isotropic magneto-thermo elastic solid half-space is investigated in [4]. The effect of magnetic Field and thermal relaxation time on three dimensional thermal shock problem in generalized thermo elasticity is studied in [5]. On Rayleigh wave in generalized magneto-thermo elastic media with hydrostatic initial stress has been illustrated in [6]. In [6], they derived governing equations of generalized magneto-thermo elasticity with hydrostatic initial stress are solved for surface wave solutions. The particular solutions in the half-space are applied to the boundary conditions at the free surface of the half-space to obtain the frequency equation of Rayleigh wave. Two temperature magneto-thermo elasticity with initial Stress: State Space Formulation is discussed in [7]. In the said paper, magneto-thermo elastic interactions in an initially stressed isotropic homogeneous elastic half-space with two temperatures are studied using mathematical methods under the purview of the L-S model of linear theory of generalized thermo elasticity. In [8] authors discussed electro-magneto-thermo-elastic plane waves in rotating media with thermal relaxation. Generation of generalized magneto thermo elastic waves by thermal shock in a perfectly conducting half-space is investigated in [9]. Mathematical modeling on rotational magneto-thermo elastic phenomenon under gravity and laser pulse considering four theories is studied in [10]. Magneto-thermo elastic plane waves in rotating media are studied in [11]. In [12] authors investigated magneto-elastic plane waves in infinite rotating media. Rotating magneto-thermo elastic rod with finite length due to moving heat sources via Eringen's nonlocal model is studied in [13]. The said paper presents an analytic solution for thermo elastic homogeneous rotating finite rod subjected to a periodic source is presented. The nonlocal governing equations of the problem were transferred by applying the Laplace transform and then were solved numerically by using Taylor's expansion series. Three Dimensional Thermal Shock Problem in Magneto-Thermo elastic Orthotropic Medium is investigated in [14]. In 14] a detail analysis of propagation of thermo elastic disturbances in an orthotropic medium in the presence of a time dependent thermal shock is studied. Two dimensional problem of magneto-thermo elasticity fiber reinforced medium under temperature-dependent properties with three-phase-lag theory is investigated in [15]. A study on fractional order magneto-thermo elasticity with three-phase-lag is studied in [16]. The problem of wave propagation in magneto-thermo elastic solids in the presence of static stresses has not been analyzed earlier and considered for the first time in this paper. The frequency equation is obtained in the presence of magneto-thermo elastic solids in the presence of static stresses. Phase velocity against wave number for different uniaxial static stress is computed for two types of materials and then discussed and presented graphically.

2. Governing Equations and Solution of the Problem:

Consider an isotropic magneto-thermo elastic solid cartesian coordinate system (x, y, z). Let $\vec{u}(u, v, w)$ are displacements. The effective stress components in the case of elastic isotropic solid is given in [17] are

$$\sigma'_{xx} = \sigma_{xx} - \sigma_{xy}\frac{\partial u}{\partial y} - \sigma_{xz}\frac{\partial u}{\partial z},$$
$$\sigma'_{xy} = \sigma_{xy} - \sigma_{xx}\frac{\partial v}{\partial x} - \sigma_{xz}\frac{\partial v}{\partial z},$$
$$\sigma'_{xz} = \sigma_{xz} - \sigma_{xx}\frac{\partial w}{\partial x} - \sigma_{xy}\frac{\partial w}{\partial y},$$
$$\sigma'_{yy} = \sigma_{yy} - \sigma_{yx}\frac{\partial v}{\partial x} - \sigma_{yz}\frac{\partial v}{\partial z},$$
$$\sigma_{yx} = \sigma_{yx} - \sigma_{yy}\frac{\partial u}{\partial y} - \sigma_{yz}\frac{\partial u}{\partial z},$$
$$\sigma'_{yz} = \sigma_{yz} - \sigma_{yx}\frac{\partial w}{\partial x} - \sigma_{yy}\frac{\partial w}{\partial y},$$
$$\sigma'_{zx} = \sigma_{zx} - \sigma_{zy}\frac{\partial u}{\partial y} - \sigma_{zz}\frac{\partial u}{\partial z},$$
$$\sigma'_{zy} = \sigma_{zy} - \sigma_{zx}\frac{\partial v}{\partial x} - \sigma_{zz}\frac{\partial v}{\partial z},$$
$$\sigma'_{zz} = \sigma_{zz} - \sigma_{zx}\frac{\partial w}{\partial x} - \sigma_{zy}\frac{\partial w}{\partial y}. \tag{1}$$

In the above usual stresses and heat equation are [18]

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \beta T)\delta_{ij},$$
$$K\nabla^2 T = \rho c_v \frac{\partial T}{\partial t} + T_0 \beta \frac{\partial e}{\partial t}. \tag{2}$$

In eq. (2), $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, σ_{ij}'s are stress components, e_{ij} are strain components, T is the change in temperature about the equilibrium T_0, ρ is the mass density, c_υ is the specific heat, β is a coupling factor that couples the heat conduction and elastic field equation, $\beta = (\frac{3\lambda + 2\mu}{3})\alpha_t$, K is the thermal conductivity, t is the time, λ and μ are Lame's constants. e is the solid dilatation. The strain e_{ij} related to the displacements are given by

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = \frac{\partial v}{\partial y}, \quad e_{zz} = \frac{\partial w}{\partial z},$$
$$\sigma_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}), \quad \sigma_{yz} = \mu(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}), \quad \sigma_{xz} = \mu(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}). \tag{3}$$

Substitution of effective stresses eq. (1) for the usual stresses eq. (2), the equations of motions takes the following form:

$$\frac{\partial}{\partial x}(\sigma'_{xx}(1 + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})) + \frac{\partial}{\partial y}(\sigma'_{xy}(1 + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z})) + \frac{\partial}{\partial z}(\sigma'_{xz}(1 + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})) + F_1 = \rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial}{\partial x}(\sigma_{yx}(1 + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})) + \frac{\partial}{\partial y}(\sigma'_{yy}(1 + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z})) + \frac{\partial}{\partial z}(\sigma'_{yz}(1 + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})) + F_2 = \rho\frac{\partial^2 v}{\partial t^2},$$

$$\frac{\partial}{\partial x}(\sigma'_{zx}(1+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}))+\frac{\partial}{\partial y}(\sigma'_{zy}(1+\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}))+\frac{\partial}{\partial z}(\sigma'_{zz}(1+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}))+F_3=\rho\frac{\partial^2 w}{\partial t^2},$$

$$K(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})T=\rho c_v\frac{\partial T}{\partial t}+\beta T_0\frac{\partial}{\partial t}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}). \quad (4)$$

In eq. (4), F_1, F_2, F_3 are the components of Lorentz forces along the x, y, z directions. Taking into the account the absence of displacement current the linearlized Maxwell equations governing the electromagnetic fields for slowly moving solid medium having electrical conductivity are [19]

$$curl\vec{h}=\vec{J},\ curl\vec{E}=-\mu_0\frac{\partial h}{\partial t},\ div\bar{h}=0,\ div\vec{E}=0,$$

$$\vec{h}=curl(\vec{u}\times H_0) \quad (5)$$

In eq. (5), $\vec{E}, \vec{J}, H_0, \mu_0, \vec{h}$ and $\vec{u}(u,v,w)$ are the electrical intensity, electric current density, primary magnetic field, magnetic permeability, perturbed magnetic field over the constant primary magnetic field vector and displacement vector respectively. Solving $\vec{J}$ of eq. (5) and then put the value of $\vec{J}$ in the equation of Lorentz force $\vec{F}=\mu_0(\vec{J}\times\vec{H}_0)$, we get the components of Lorentz force as

$$F_1=\mu_0 H_0^2(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}),\ F_2=\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}),\ F_3=\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial z^2}). \quad (6)$$

Eq. (3) together with eq. (1) and eq. (6) must satisfy at every inside and on the surface of the body. Boundary conditions are to be imposed to solve the problem completely. To construct the relationship between the time variant and static elastic quantities, the particular displacements and stresses are given in [17]

$$u=\sum u_n(r,\theta,z,\omega_n,t)=u_0+u_1+u_2+\cdots,$$
$$v=\sum v_n(r,\theta,z,\omega_n,t)=v_0+v_1+v_2+\cdots,$$
$$w=\sum w_n(r,\theta,z,\omega_n,t)=w_0+w_1+w_2+\cdots,$$
$$\sigma_{ij}=\sum\sigma_{ij_n}(r,\theta,z,\omega_n,t)=\sigma_{ij_0}+\sigma_{ij_1}+\sigma_{ij_2}+\cdots. \quad (7)$$

In eq. (7), ω_n is the n^{th} angular frequency. Using the equations (1), (2), (6), (7) in (4) we get the static equations $(n=0)$ given below.

$$\frac{\partial}{\partial x}((\sigma_{xx_0}-\sigma_{xy_0}\frac{\partial u_0}{\partial y}-\sigma_{xz_0}\frac{\partial u_0}{\partial z})(1+\frac{\partial v_0}{\partial y}+\frac{\partial w_0}{\partial z}))+\frac{\partial}{\partial y}((\sigma_{xy_0}-\sigma_{xx_0}\frac{\partial v_0}{\partial x}-\sigma_{xz_0}\frac{\partial v_0}{\partial z})(1+\frac{\partial u_0}{\partial x}+\frac{\partial w_0}{\partial z}))+$$
$$\frac{\partial}{\partial z}((\sigma_{xz_0}-\sigma_{xx_0}\frac{\partial w_0}{\partial x}-\sigma_{xy_0}\frac{\partial w_0}{\partial y})(1+\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z})=0,$$
$$\frac{\partial}{\partial x}((\sigma_{yx_0}-\sigma_{yy_0}\frac{\partial u_0}{\partial y}-\sigma_{yz_0}\frac{\partial u_0}{\partial z})(1+\frac{\partial v_0}{\partial y}+\frac{\partial w_0}{\partial z}))+\frac{\partial}{\partial y}((\sigma_{yy_0}-\sigma_{yX_0}\frac{\partial v_0}{\partial x}-\sigma_{yz_0}\frac{\partial v_0}{\partial z})(1+\frac{\partial u_0}{\partial x}+\frac{\partial w_0}{\partial z}))+$$
$$\frac{\partial}{\partial z}((\sigma_{yz_0}-\sigma_{yy_0}\frac{\partial w_0}{\partial y}-\sigma_{yx_0}\frac{\partial w_0}{\partial x})(1+\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z})=0,$$
$$\frac{\partial}{\partial x}((\sigma_{zx_0}-\sigma_{zz_0}\frac{\partial u_0}{\partial z}-\sigma_{zy_0}\frac{\partial u_0}{\partial\theta})(1+\frac{\partial v_0}{\partial y}+\frac{\partial w_0}{\partial z}))+\frac{\partial}{\partial y}((\sigma_{zy_0}-\sigma_{zz_0}\frac{\partial v_0}{\partial z}-\sigma_{zx_0}\frac{\partial v_0}{\partial x})(1+\frac{\partial u_0}{\partial x}+\frac{\partial w_0}{\partial z}))+$$
$$\frac{\partial}{\partial z}((\sigma_{zz_0}-\sigma_{zy_0}\frac{\partial w_0}{\partial y}-\sigma_{zx_0}\frac{\partial w_0}{\partial x})(1+\frac{\partial v_0}{\partial y}+\frac{\partial u_0}{\partial x}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial z^2})=0,$$
$$K(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})T=\rho c_v\frac{\partial T}{\partial t}+\beta T_0\frac{\partial}{\partial t}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}). \quad (8)$$

Similarly, the harmonic equations $(n=1)$ are as follows

$$\frac{\partial}{\partial r}((\sigma_{xx_0}-\sigma_{xy_1}\frac{\partial u_o}{\partial y}-\sigma_{xy_o}\frac{\partial u_1}{\partial y}-\sigma_{xz_1}\frac{\partial u_o}{\partial z}-\sigma_{xz_0}\frac{\partial u_1}{\partial z})(1+\frac{\partial v_o}{\partial y}+\frac{\partial w_o}{\partial z}))+((\sigma_{xx_o}-\sigma_{xy_1}\frac{\partial u_o}{\partial y}-\sigma_{xz_o}\frac{\partial u_o}{\partial z})$$
$$\times(\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}))+\frac{\partial}{\partial y}((\sigma_{xy_1}-\sigma_{xx_1}\frac{\partial v_o}{\partial x}-\sigma_{xx_o}\frac{\partial v_1}{\partial x}-\sigma_{xz_1}\frac{\partial v_o}{\partial z}-\sigma_{xz_o}\frac{\partial v_1}{\partial z})(1+\frac{\partial u_o}{\partial x}+\frac{\partial w_o}{\partial z}))+(\sigma_{xy_o}-\sigma_{xx_o}$$
$$\times\frac{\partial v_o}{\partial x}-\sigma_{xz_o}\frac{\partial v_o}{\partial z})(\frac{\partial u_1}{\partial x}+\frac{\partial w_1}{\partial z}))+\frac{\partial}{\partial z}((\sigma_{xz_1}-\sigma_{xx_1}\frac{\partial w_o}{\partial x}-\sigma_{xx_o}\frac{\partial w_1}{\partial x}-\sigma_{xy_1}\frac{\partial w_0}{\partial y}-\sigma_{xy_0}\frac{\partial w_1}{\partial y})(1+\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}))$$
$$+(\sigma_{xz_0}-\sigma_{xx_0}\frac{\partial w_0}{\partial x}-\sigma_{xy_0}\frac{\partial w_0}{\partial y})(\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z})=\frac{\partial^2 u_1}{\partial t^2},$$

$$\frac{\partial}{\partial x}((\sigma_{yx_1}-\sigma_{yy_1}\frac{\partial u_0}{\partial y}-\sigma_{yy_0}\frac{\partial u_1}{\partial y}-\sigma_{yz_1}\frac{\partial u_0}{\partial z}-\sigma_{yz_1}\frac{\partial u_1}{\partial z})(1+\frac{\partial v_0}{\partial y}+\frac{\partial w_0}{\partial z})+(\sigma_{yx_0}-\sigma_{yy_0}\frac{\partial u_0}{\partial y}-\sigma_{yz_0}\frac{\partial u_0}{\partial z})$$
$$\times(\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}))+\frac{\partial}{\partial y}((\sigma_{yy_1}-\sigma_{yx_1}\frac{\partial v_0}{\partial x}-\sigma_{yx_0}\frac{\partial v_1}{\partial x}-\sigma_{yz_1}\frac{\partial v_0}{\partial z}-\sigma_{yz_0}\frac{\partial v_1}{\partial z})(1+\frac{\partial u_0}{\partial x}+\frac{\partial w_0}{\partial z}))+((\sigma_{yy_0}-\sigma_{yx_0}\frac{\partial v_0}{\partial x}$$
$$-\sigma_{rz_0}\frac{\partial v_0}{\partial z})(\frac{\partial u_1}{\partial x}+\frac{\partial w_1}{\partial z}))+\frac{\partial}{\partial z}((\sigma_{yz_1}-\sigma_{yy_1}\frac{\partial w_0}{\partial y}-\sigma_{yy_0}\frac{\partial w_1}{\partial y}-\sigma_{yx_1}\frac{\partial w_0}{\partial x}-\sigma_{yx_0}\frac{\partial w_1}{\partial x})\times(1+\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y})+(\sigma_{yz_0}$$
$$-\sigma_{yy_0}\frac{\partial w_0}{\partial y}-\sigma_{yx_0}\frac{\partial w_0}{\partial x})(\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z})=\rho\frac{\partial^2 v_1}{\partial t^2},$$

$$\frac{\partial}{\partial x}((\sigma_{zx_{11}}-\sigma_{zz_1}\frac{\partial u_0}{\partial z}-\sigma_{zz_0}\frac{\partial u_1}{\partial z}-\sigma_{zy_1}\frac{\partial u_0}{\partial y}-\sigma_{zy_0}\frac{\partial u_1}{\partial y})(1+\frac{\partial v_0}{\partial y}+\frac{\partial w_0}{\partial z})+(\sigma_{zx_0}-\sigma_{zz_0}\frac{\partial u_0}{\partial z}$$
$$-\sigma_{zy_0}\frac{\partial u_0}{\partial \theta})(\frac{\partial w_1}{\partial z}+\frac{\partial v_1}{\partial y}))+\frac{\partial}{\partial y}((\sigma_{zy_1}-\sigma_{zz_1}\frac{\partial v_0}{\partial z}-\sigma_{zz_0}\frac{\partial v_1}{\partial z}-\sigma_{zx_1}\frac{\partial v_0}{\partial x}-\sigma_{zx_0}\frac{\partial v_1}{\partial x})(1+\frac{\partial u_0}{\partial x}+\frac{\partial w_0}{\partial z})$$
$$+(\sigma_{zy_0}-\sigma_{zz_0}\frac{\partial v_0}{\partial z}-\sigma_{zx_0}\frac{\partial v_0}{\partial x})(\frac{\partial u_1}{\partial x}+\frac{\partial w_1}{\partial z}))+\frac{\partial}{\partial z}((\sigma_{zz_1}-\sigma_{zy_1}\frac{\partial w_0}{\partial y}-\sigma_{zy_0}\frac{\partial w_1}{\partial y}-\sigma_{zx_1}\frac{\partial w_0}{\partial x}-\sigma_{zx_0}\frac{\partial w_1}{\partial x})$$
$$\times(1+\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y})+(\sigma_{zz_0}-\sigma_{zy_0}\frac{\partial w_0}{\partial y}-\sigma_{zx_0}\frac{\partial w_0}{\partial x})(\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial z^2})=\rho\frac{\partial^2 w_1}{\partial t^2}$$
$$K(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})T=\rho c_v\frac{\partial T}{\partial t}+\beta T_0\frac{\partial}{\partial t}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}). \quad (9)$$

The eqs. (8) and (9) are the static equations and first harmonic equations, respectively. In eq. (9) the suffix one on the displacement components and the stresses has been dropped.

3. Equation of Uniaxial Static Stress:

It is assume that applied static uniaxial stress is acting in the direction of z -axis, then we have

$$\sigma_{ij_0}=e_{ij_0}=0 \quad (i\neq j),$$
$$\sigma_{xx_0}=\sigma_{yy_0}=0, \quad (10)$$

where σ_{zz_0} is static uniaxial stress. Substituting the eqs. (10) in the eqs. (9), we can see that the eqs. (8) automatically satisfied and the static strains are obtained as follows:

$$e_{xx_0}=e_{yy_0}=-\vartheta\, e_{zz_0}, \quad T=T_0. \quad (11)$$

Substituting the above eqs.(10)and (11) in eq. (9) we the get the equations in the following form

$$\left[1+(1-\vartheta)\frac{\partial w_0}{\partial z}\right]\left[\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}\right]+\left[1-2\vartheta\frac{\partial w_0}{\partial z}\right]\frac{\partial\sigma_{xz}}{\partial z}+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z})=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\left[1+(1-\vartheta)\frac{\partial w_0}{\partial z}\right]\left[\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}\right]+\left[1-2\vartheta\frac{\partial w_0}{\partial z}\right]\frac{\partial\sigma_{yz}}{\partial z}+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z})=\rho\frac{\partial^2 v}{\partial t^2},$$

$$\left[1+(1-\vartheta)\frac{\partial w_0}{\partial z}\right]\left[\frac{\partial \sigma_{zx}}{\partial x}+\frac{\partial \sigma_{zy}}{\partial y}\right]+\left[1-2\vartheta\frac{\partial w_0}{\partial z}\right]\frac{\partial \sigma_{zz}}{\partial z}+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial z^2})=\rho\frac{\partial^2 w}{\partial t^2},$$

$$K(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2})=\rho c_v\frac{\partial T}{\partial t}+\beta T_0(\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^2 v}{\partial y\partial t}+\frac{\partial^2 w}{\partial z\partial t}). \quad (12)$$

Substituting equations (11) in the first equation of the eq. (2), we obtain the following equation

$$e_{zz_0}=\frac{\sigma_{zz_0}+\beta T_0}{Y}. \quad (13)$$

In the above, $\vartheta=\dfrac{\lambda}{2(\lambda+\mu)}$ is the Poisson's ratio and $Y=\dfrac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}$ is the Young's modulus. Substituting the eqs. (2), (3), (13) in eq. (12), the equations of motion are given below

$$\mu\nabla^2 u+(\mu+\lambda)\frac{\partial e}{\partial x}-\beta\frac{\partial T}{\partial x}++\frac{1}{2(\lambda+\mu)}(\frac{\sigma_{zz_0}+\beta T_0}{Y})((\lambda+2\mu)(\lambda\frac{\partial^2 u}{\partial x^2}+2\mu\frac{\partial^2 u}{\partial x^2}+\lambda\frac{\partial^2 v}{\partial x\partial y}+$$

$$\lambda\frac{\partial^2 w}{\partial x\partial y}-\beta\frac{\partial T}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}+\mu\frac{\partial^2 v}{\partial x\partial y})-(2\lambda\mu\frac{\partial^2 u}{\partial z^2}+2\lambda\mu\frac{\partial^2 w}{\partial z\partial x}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z})=\rho\frac{\partial^2 u}{\partial t^2},$$

$$\mu\nabla^2 v+(\mu+\lambda)\frac{\partial e}{\partial y}-\beta\frac{\partial T}{\partial y}++\frac{1}{2(\lambda+\mu)}(\frac{\sigma_{zz_0}+\beta T_0}{Y})((\lambda+2\mu)(\mu\frac{\partial^2 v}{\partial x^2}+2\mu\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 u}{\partial x\partial y}+$$

$$\lambda\frac{\partial^2 u}{\partial x\partial y}-\beta\frac{\partial T}{\partial x}+\lambda\frac{\partial^2 v}{\partial y^2}+\lambda\frac{\partial^2 w}{\partial z\partial y})-(2\lambda\mu\frac{\partial^2 v}{\partial z^2}+2\lambda\mu\frac{\partial^2 w}{\partial y\partial z}))+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z})=\rho\frac{\partial^2 v}{\partial t^2},$$

$$\mu\nabla^2 w+(\mu+\lambda)\frac{\partial e}{\partial z}-\beta\frac{\partial T}{\partial z}+\frac{1}{2(\lambda+\mu)}(\frac{\sigma_{zz_0}+\beta T_0}{Y})((\lambda+2\mu)(\mu\frac{\partial^2 w}{\partial z^2}+\mu\frac{\partial^2 v}{\partial y\partial z}+\mu\frac{\partial^2 u}{\partial x\partial z}+$$

$$\mu\frac{\partial^2 w}{\partial y^2})-2\lambda(2\mu\frac{\partial^2 w}{\partial z^2}+\lambda\frac{\partial^2 u}{\partial x\partial z}+\lambda\frac{\partial^2 v}{\partial y^2}+\lambda\frac{\partial^2 w}{\partial y^2}-\beta\frac{\partial T}{\partial z}))+\sigma_{zz_0}(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial y\partial z})+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial z}+$$

$$\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial z^2})=\rho\frac{\partial^2 w}{\partial t^2},$$

$$K(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2})=\rho c_v\frac{\partial T}{\partial t}+\beta T_0(\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^2 v}{\partial y\partial t}+\frac{\partial^2 w}{\partial z\partial t}). \quad (14)$$

The solution of physical variables of eq. (14) can be decomposed in the following form:

$$(u,v,w,T)(x,y,z,t)=(C_1,C_2,C_3,C_4)e^{j\omega t-j(k_1x+k_2y+k_3z)}. \quad (15)$$

In the above, C_1,C_2,C_3,C_4 are arbitrary constants, k_i $(i=1,2,3)$ is the wave number in the j^{th} direction, such that the wave number $k=\sqrt{k_1^2+k_2^2+k_3^2}$. substituting the eq. (15) in the eq. (14), the equations of motion in terms of displacement components are as follows:

$$(2\mu j^2k_1^2+\mu j^2k_2^2+\mu j^2k_3^2+\lambda_1 j^2k_1^2+(\frac{\sigma_{zz_0}+\beta T_0}{2(\lambda+\mu)Y})(\lambda+2\mu)(2\mu j^2k_1^2+\lambda k_1^2j^2+\mu k_2^2j^2)-2\lambda\mu k_3^2j^2$$

$$+\mu_0H_0^2k_1^2j^2-\rho\omega^2j^2)C_1+(\lambda k_1k_2j^2+\mu k_1k_2j^2+(\frac{\sigma_{zz_0}+\beta T_0}{2(\lambda+\mu)Y})(\lambda k_1k_2j^2+\mu k_1k_3j^2)+\mu_0H_0^2k_1k_2j^2)C_2$$

$$+(\lambda k_1k_3+\mu k_1k_3+(\frac{\sigma_{zz_0}+\beta T_0}{2(\lambda+\mu)Y})\lambda k_1k_3+\frac{2\lambda\mu}{\lambda+\mu}k_1k_3+\mu_0H_0^2k_1k_3)j^2C_3-(\beta k_1j+(\frac{\sigma_{zz_0}+\beta T_0}{2(\lambda+\mu)Y})\beta k_1j))C_4=0,$$

$$(\mu k_1 k_2 j^2 + \lambda k_1 k_2 j^2 + (\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y})(\lambda+2\mu)(\mu k_1 k_2 j^2 + \lambda k_1 k_2 j^2) + \mu_0 H_0^2 k_1 k_2 j^2)C_1 + (\mu j^2 (k_1^2 + 2k_2^2 +$$

$$k_3^2) + \lambda k_2^2 j^2 + (\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y})(2\lambda\mu k_1^2 + 4\lambda\mu k_2^2 - 4\lambda^2 \mu k_3^2) - \rho\omega^2 + \mu_0 H_0^2 k_2^2) j^2 C_2 + ((\mu+\lambda)k_3 k_2$$

$$+(\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y}) \times (\lambda+2\mu)(\lambda k_3 k_2 - 2\lambda\mu k_3 k_2) + \mu_0 H_0^2 k_3 k_2) j^2 C_3 - (\beta k_2 + (\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y}) \times (\lambda+2\mu)\beta k_2) jC_4 = 0,$$

$$((\mu+\lambda) j^2 k_1 k_3 + (\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y})(\lambda+2\mu)(\mu-2\lambda) j^2 k_1 k_3 + (\sigma_{zz_0} + \mu_0 H_0^2) j^2 k_1 k_3)C_1 +$$

$$((\mu+\lambda) j^2 k_2 k_3 + (\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y})(\lambda+2\mu)(\mu-2\lambda) j^2 k_2 k_3 + (\sigma_{zz_0} + \mu_0 H_0^2) j^2 k_2 k_3)C_2 +$$

$$((\mu j^2 (k_1^2 + k_2^2) + (2\mu+\lambda) j^2 k_3^2 + \mu j^2 k_2 k_3 + (\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y})(\lambda+2\mu)(\mu k_1^2 + \mu k_2^2) j^2 -$$

$$(4\lambda\mu + 2\lambda^2) j^2 k_3^2 + \mu_0 H_0^2 j^2 k_3^2 - \rho\omega^2)C_3 - (\beta j\rho\omega - (\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y})(2\lambda\beta\omega j))C_4 = 0,$$

$$T_0 \beta j^2 \omega k_1 C_1 + T_0 \beta j^2 \omega k_2 C_2 + T_0 \beta j^2 \omega k_3 C_3 + K((k_1^2 + k_2^2 + k_3^2) j^2 + jc_v \omega)C_4 = 0. \quad (16)$$

4. Numerical Results and Discussion:

For the sake of numerical results the wave propagation is consider along z-direction. In this case $k_1 = k_2 = 0$. Then the equations of motion reduces to

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = 0. \quad (17)$$

Where

$$B_{11} = (\mu - \frac{\lambda(\sigma_{zz_0} + \beta T_0)}{3\lambda + 2\mu} - \rho c^2),$$

$$B_{22} = (\mu - \frac{\lambda(\sigma_{zz_0} + \beta T_0)}{3\lambda + 2\mu} - \rho c^2),$$

$$B_{33} = (2\mu + \lambda - \frac{\lambda(\sigma_{zz_0} + \beta T_0)}{2\mu(3\lambda + 2\mu)}(4\lambda\mu + 2\lambda^2 - \rho c^2)) + \mu_0 H_0^2,$$

$$B_{34} = \tfrac{1}{j}(\beta\omega + \frac{\lambda\beta\rho\omega(\sigma_{zz_0} + \beta T_0)}{3\lambda\mu + 2\mu^2}),$$

$$B_{43} = -T_0 \beta c^2, \quad B_{44} = c_v jc - K,$$

$$B_{12}, B_{13}, B_{14}, B_{21}, B_{23}, B_{24}, B_{31}, B_{32}, B_{41}, B_{42} = 0. \quad (18)$$

In the above eq. (18), $c = \dfrac{\omega}{k}$, c is the phase velocity and $\beta = (\dfrac{3\lambda + 2\mu}{3})\alpha_t$. For a non-trivial solution, the determinant of above coefficient matrix is zero. This leads to the frequency equation:

$$\begin{vmatrix} B_{11} & 0 & 0 & 0 \\ 0 & B_{22} & 0 & 0 \\ 0 & 0 & B_{33} & B_{34} \\ 0 & 0 & B_{43} & B_{44} \end{vmatrix} = 0. \qquad (19)$$

In the above eq. (19) we have calculated real phase velocity. Here imaginary part is neglected. In order to illustrate theoretical results obtained the proceeding sections; we now present some numerical results. Materials choosen for this purpose are Magnesium, Copper is given in [7, 10].The physical data are given below:

Magnesium:

$$\lambda = 9.4\times10^{10}\,Nm^{-2},\ \mu = 4.0\times10^{10}\,Nm^{-2}, \rho = 1.74\times10^{3}\,Kgm^{-3}, c_v = 1.04\times10^{3}\,JKg^{-1}\deg^{-3},$$

$$T_0 = 298K,\ K = 1.7\times10^{2}Wm^{-1}K^{-1},\ \alpha_t = 2.68\times10^{6}\,Nm^{-2}\deg^{-1}, \mu_0 = 4\pi\times10^{-7}\,Hm^{-1}, .H_0 = \frac{10^{7}}{4\pi}Am^{-1}.$$

Copper:

$$\lambda = 7.76\times10^{10}\,Nm^{-2},\ \mu = 3.86\times10^{10}\,Nm^{-2}, \rho = 8954\times10^{3}\,Kgm^{-3}, c_v = 383.1JKg^{-1}\deg^{-3},$$

$$T_0 = 293K,\ K = 386Wm^{-1}K^{-1},\ \alpha_t = 1.78\times10^{-5}\,K^{-1}, \mu_0 = 4\pi\times10^{-3}\,Hm^{-1}, .H_0 = 9\times10^{5}\,Am^{-1}.$$

Applying these parameter values in Eq. (19), the implicit relation between the phase velocity and wave number for fixed static uniaxial stress is obtained. Phase velocity is computed against wave number for static uniaxial stress. The values are computed using the bisection method implemented in MATLAB, and the results are depicted in figures 1-3. Figure.1-3 shows the plots of phase velocity against the wave number for (static stress) ST=100,200,300 in the case of magnesium, copper materials. From the figure 1-3, phase velocity increases with increase in wave number. And also as static uniaxial stress and wave number increases phase velocity increases. In general, copper material values are greater than magnesium. But the phase velocity values are higher in the case of magnesium material than copper material. This difference is due to the presence of magnetic permeability.

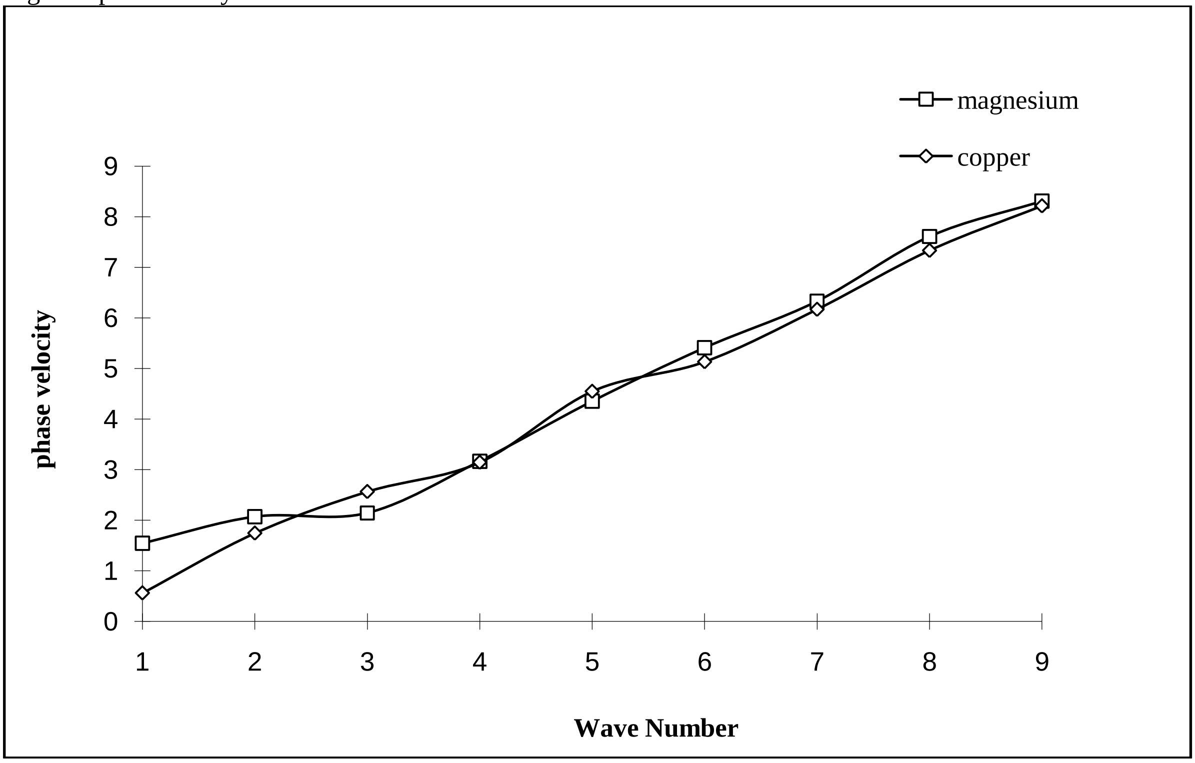

Figure 1: Variation of phase velocity with the wave number at ST-100

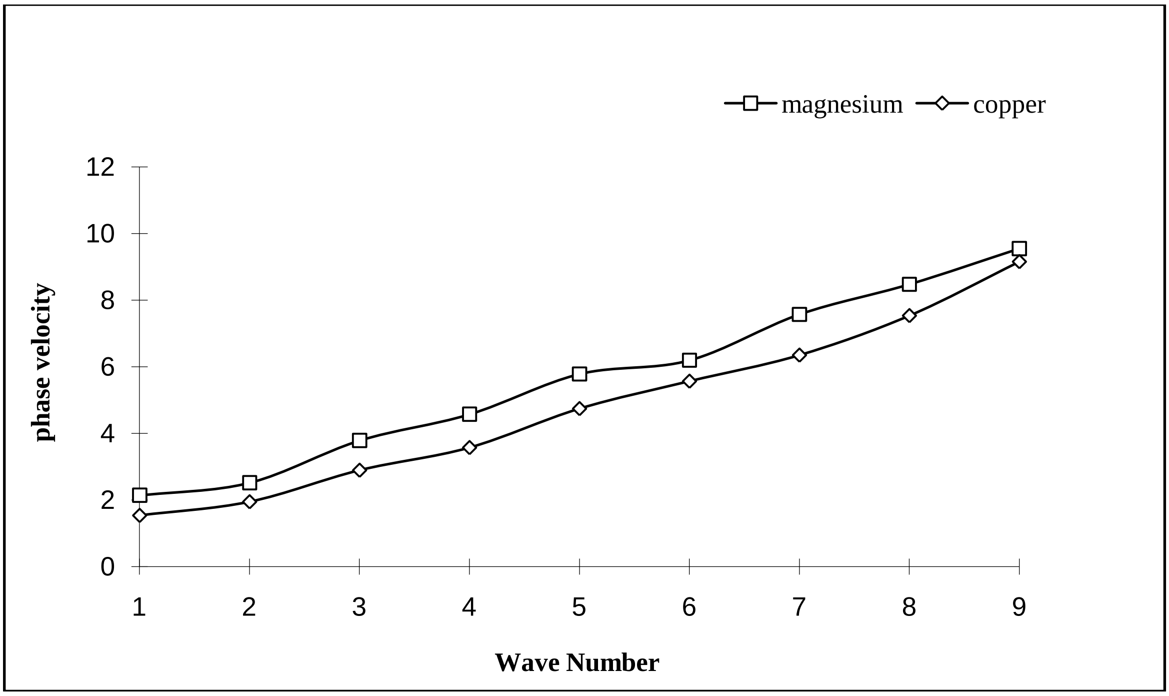

Figure 2: Variation of phase velocity with the wave number at ST-200

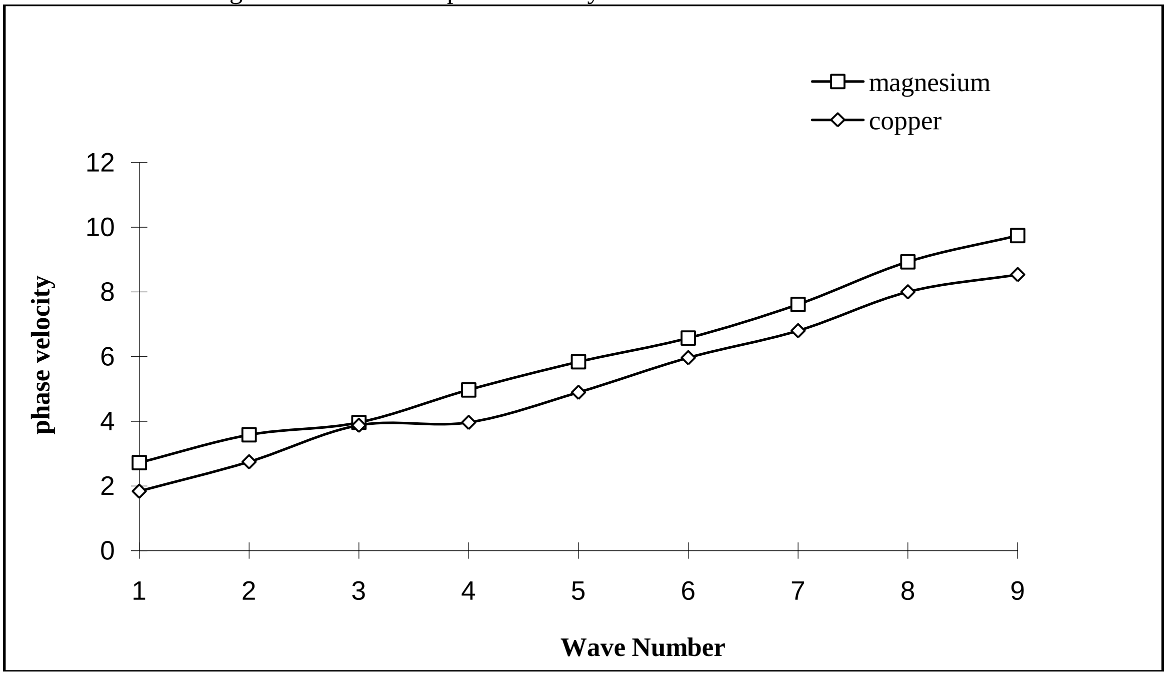

Figure 3: Variation of phase velocity with the wave number at ST-300

Conclusion:

Wave propagation in magneto-thermo elastic solids in the presence of static stresses is investigated. The frequency equation is obtained in the presence of magneto-thermo elastic solids in the presence of static stresses. Phase velocity against wave number for different uniaxial static stress is computed for two types of materials is discussed and presented graphically. In all the cases phase velocity increases with increase in wave number and static uniaxial stress.

Conflict of Interest:

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Nomenclature:

λ, μ Lames constants

T_0 Reference temperature

ρ density

c_v specific heat

K Thermal conductivity

α_t coefficient of linear thermal expansion

$\beta = (\frac{3\lambda + 2\mu}{3})\alpha_t$

t time

σ_{ij} components of stress

e_{ij} components of strains

δ_{ij} Kroneckar delta

e cubical dilatation

μ_0 Magnetic permeability

E Electric displacement

J current density vector

H Total magnetic intensity vector

H_0 Initial uniform magnetic field

F_i Components of Lorentz body force

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